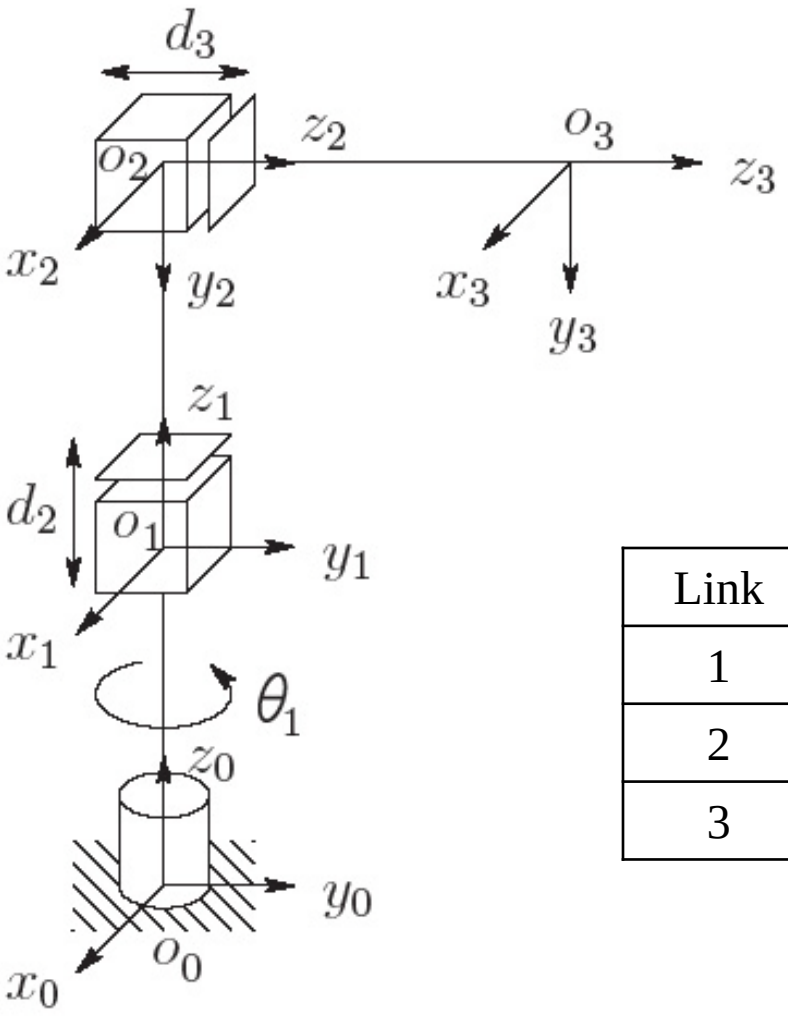
Day 09

More Denavit-Hartenberg Examples

Step 5: Find the DH parameters



Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1*
2	0	-90	d_2*	0
3	0	0	d_3*	0

* joint variable

Figure 3.7: Three-link cylindrical manipulator.

Step 6: Compute the transformation

- once the DH parameters are known, it is easy to construct the overall transformation

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1*
2	0	-90	d_2*	0
3	0	0	d_3*	0

* joint variable

$$T_1^0 = R_{z,\theta_1} T_{z,d_1} T_{x,a_1} R_{x,\alpha_1} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 6: Compute the transformation

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1*
2	0	-90	d_2*	0
3	0	0	d_3*	0

* joint variable

$$T\,{}^{1}_{2} = R_{z,\theta_2} T_{z,d_2} T_{x,a_2} R_{x,\alpha_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 6: Compute the transformation

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1*
2	0	-90	d_2*	0
3	0	0	d_3*	0

* joint variable

$$T_3^2 = R_{z,\theta_3} T_{z,d_3} T_{x,a_3} R_{x,\alpha_3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 6: Compute the transformation

$$T_3^0 = T_1^0 T_2^1 T_3^2 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spherical Wrist

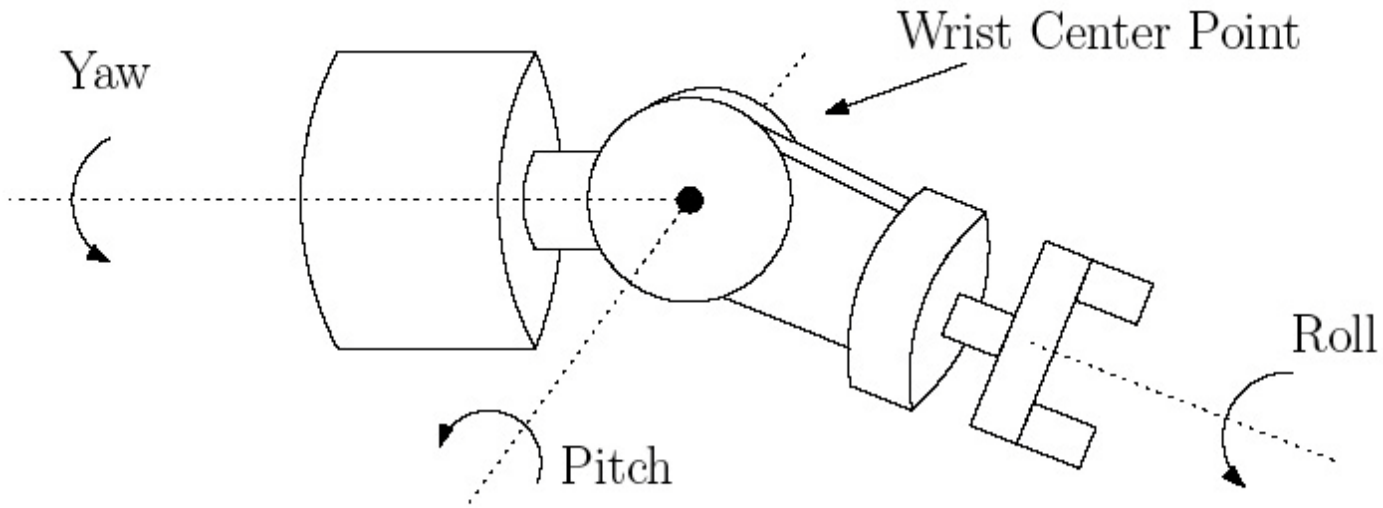
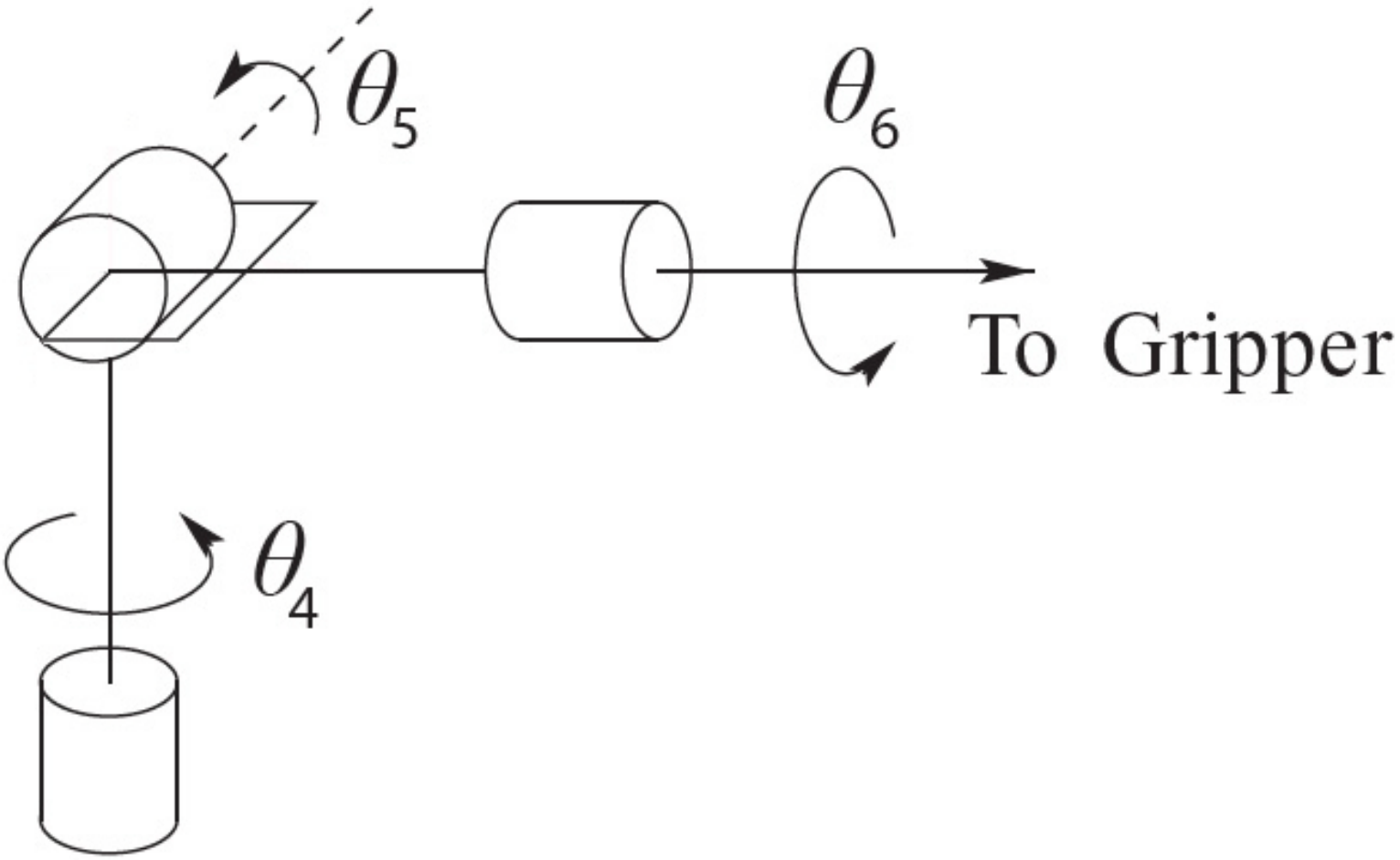
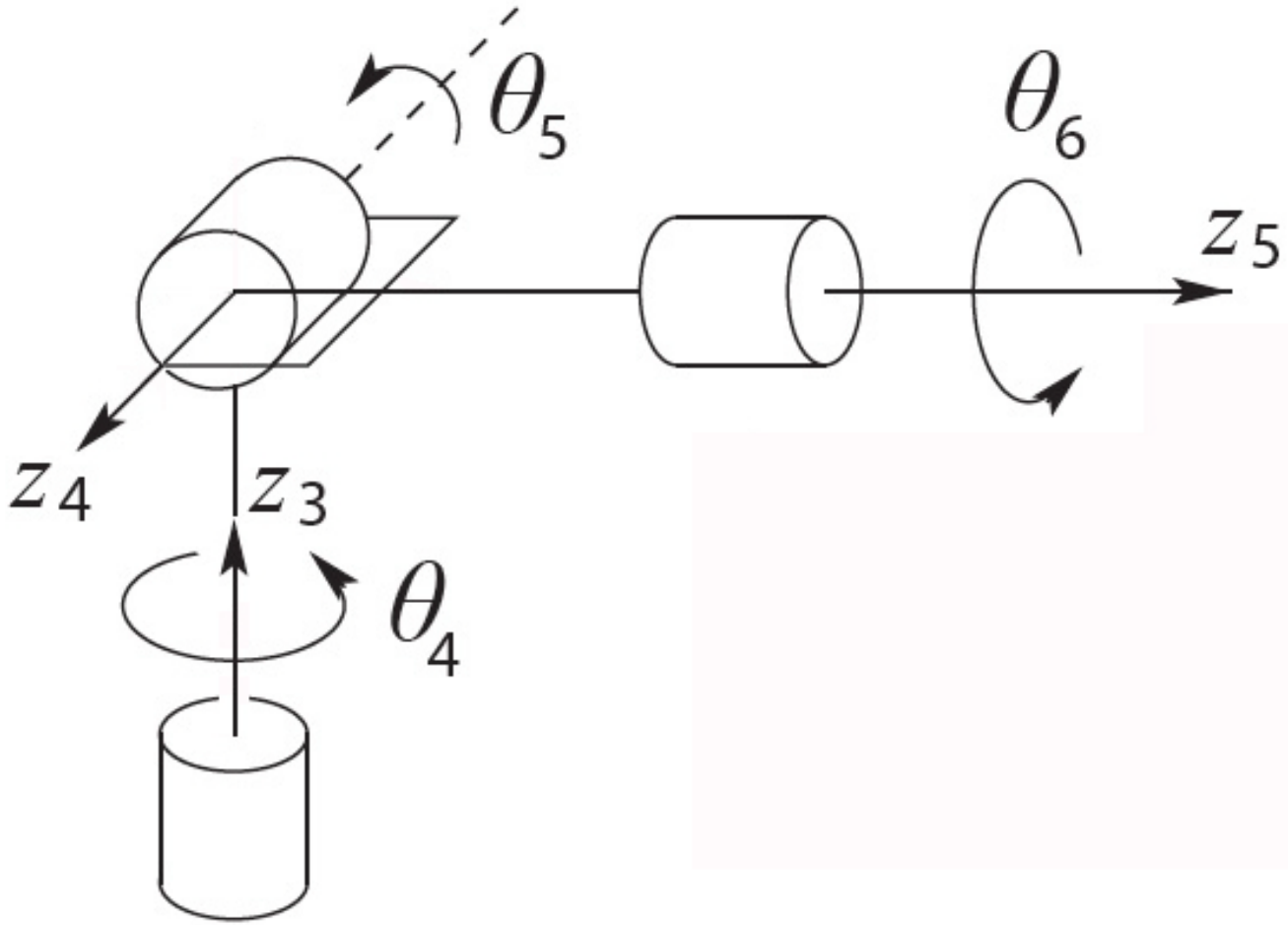


Figure 1.6: The spherical wrist. The axes of rotation of the spherical wrist are typically denoted roll, pitch, and yaw and intersect at a point called the wrist center point.

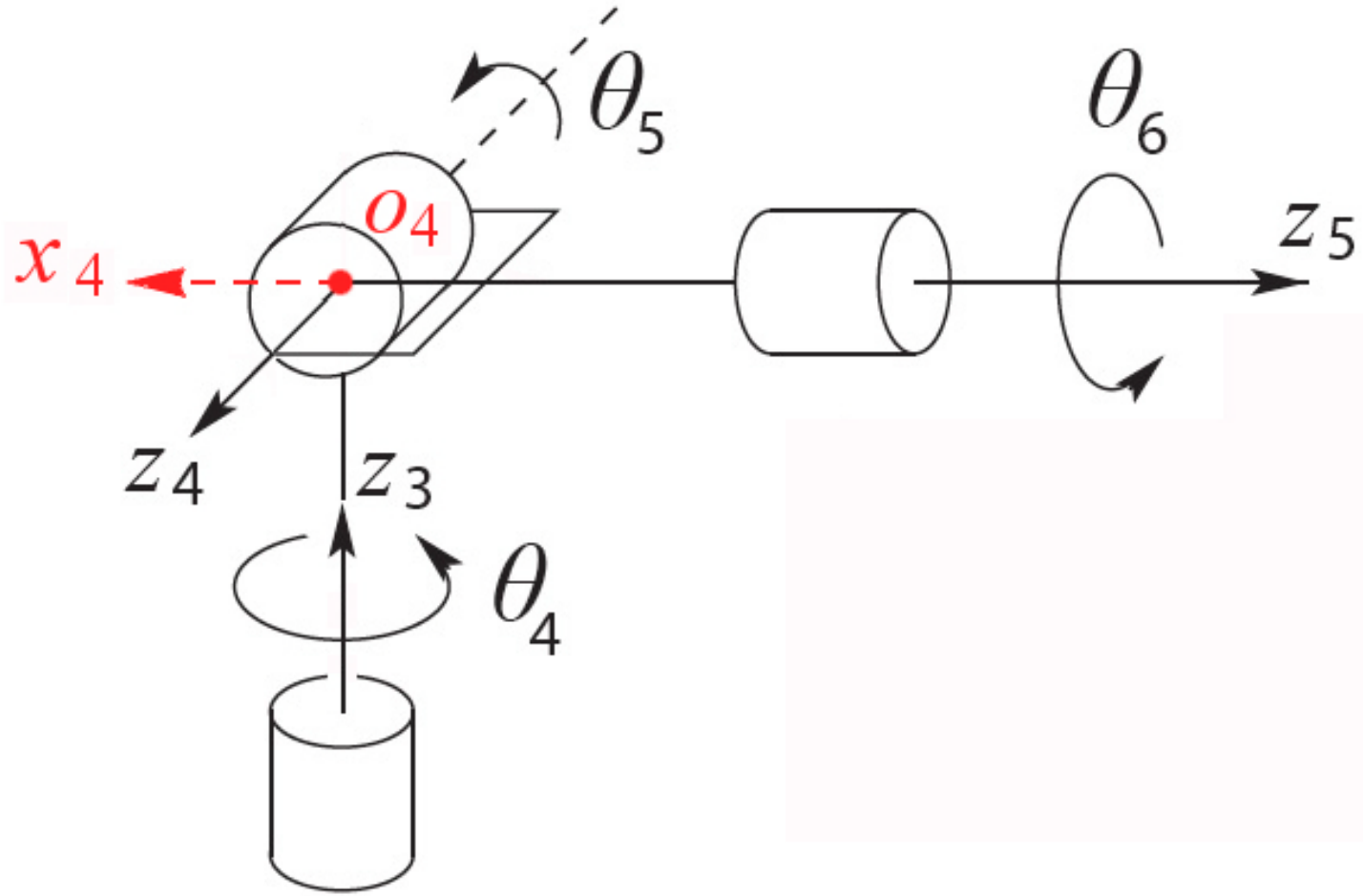
Spherical Wrist



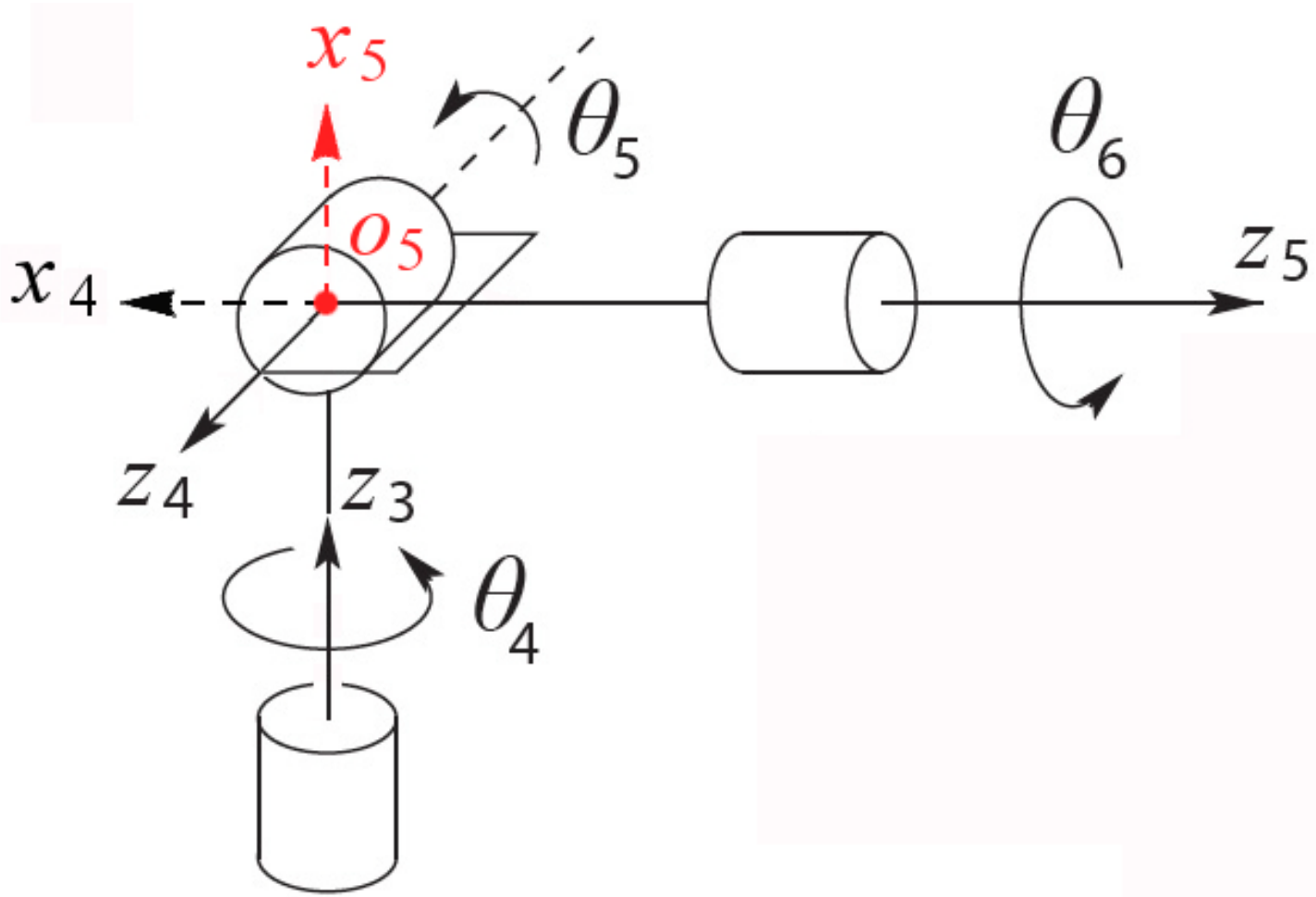
Spherical Wrist: Step 1



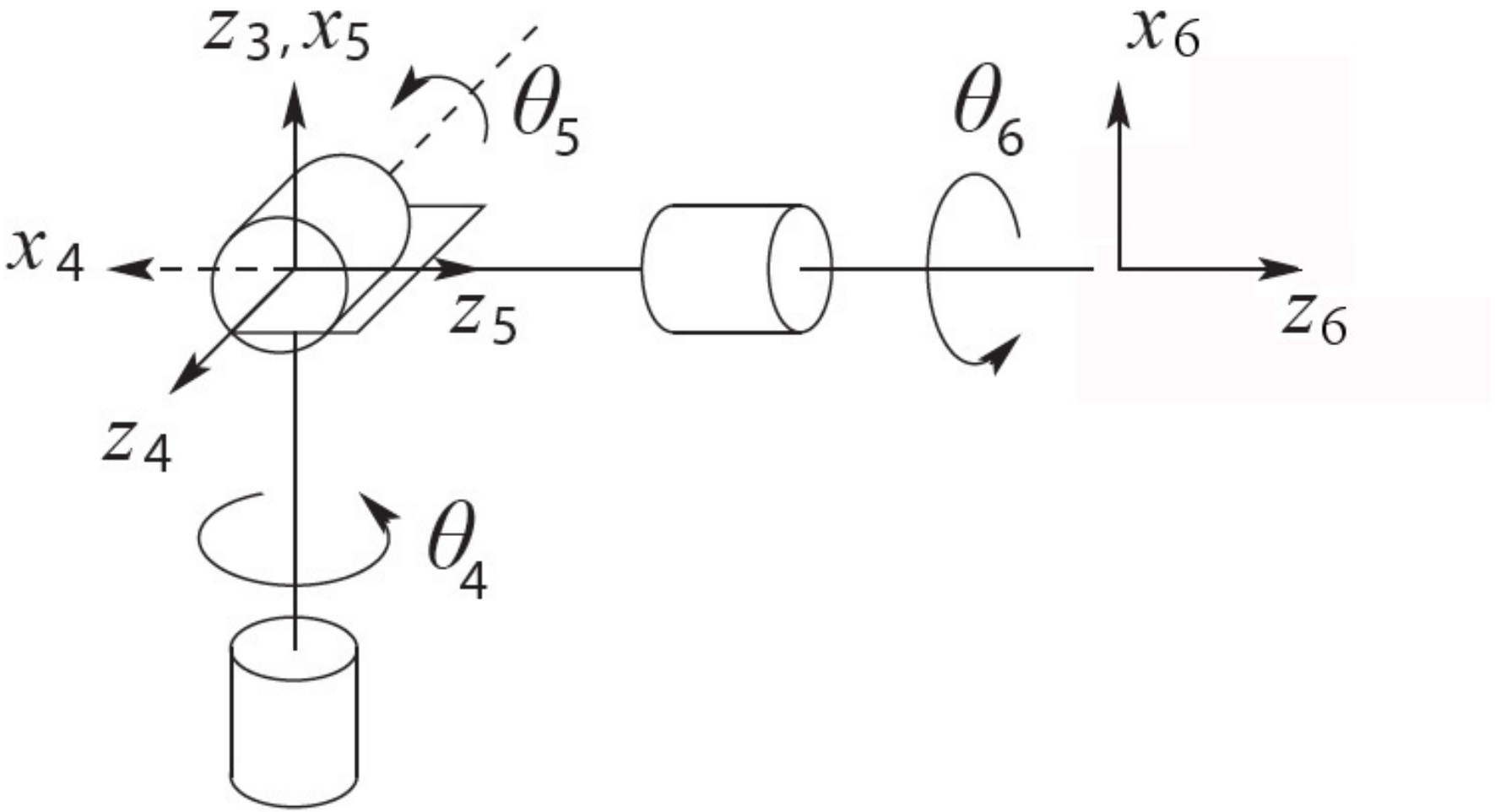
Spherical Wrist: Step 2



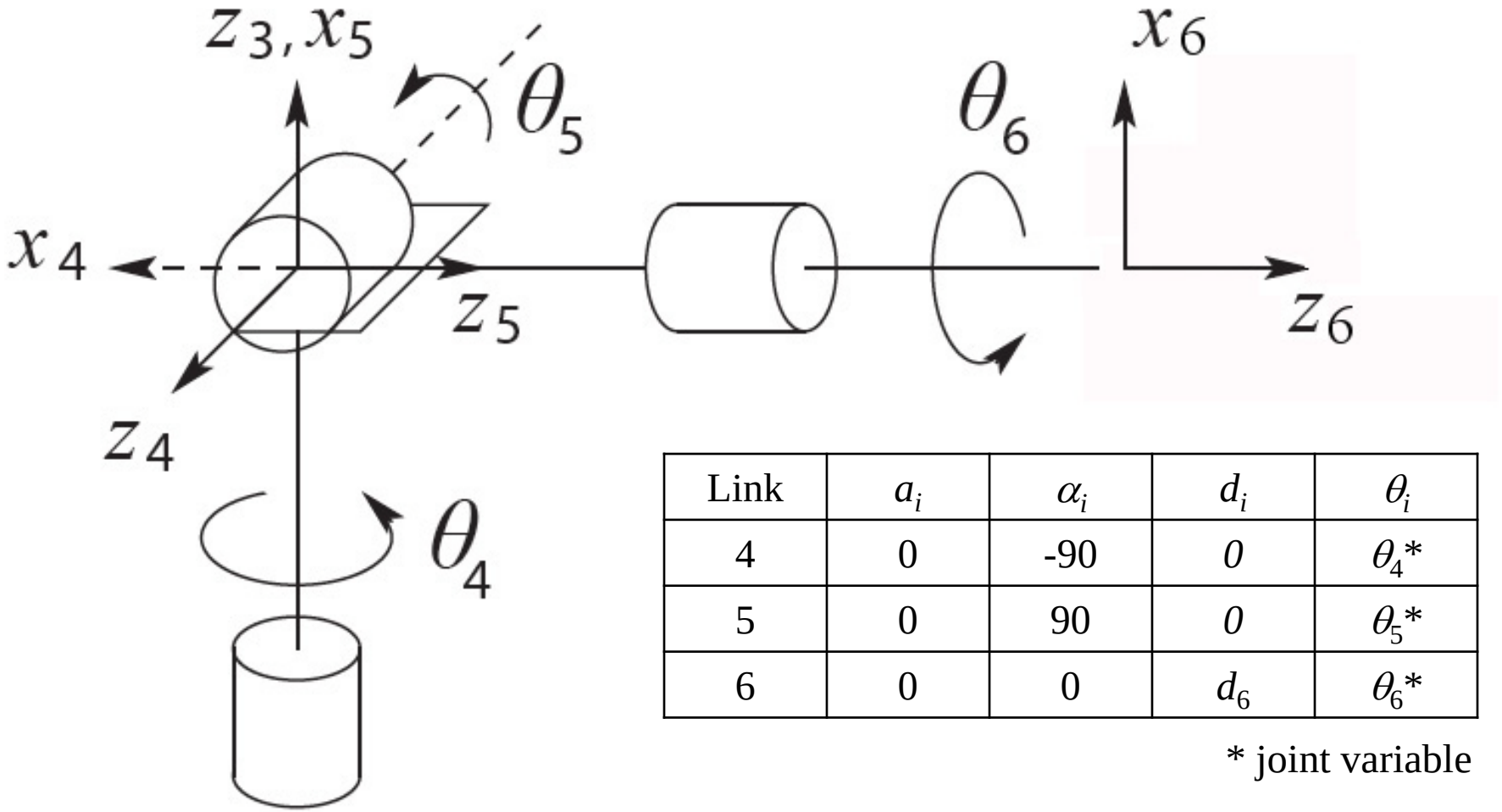
Spherical Wrist: Step 2



Spherical Wrist: Step 4



Step 5: DH Parameters



Link	a_i	α_i	d_i	θ_i
4	0	-90	*0*	θ_4*
5	0	90	*0*	θ_5*
6	0	0	d_6	θ_6*

* joint variable

Step 6: Compute the transformation

$$T_6^3 = T_4^3 T_5^4 T_6^5 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

RPP + Spherical Wrist

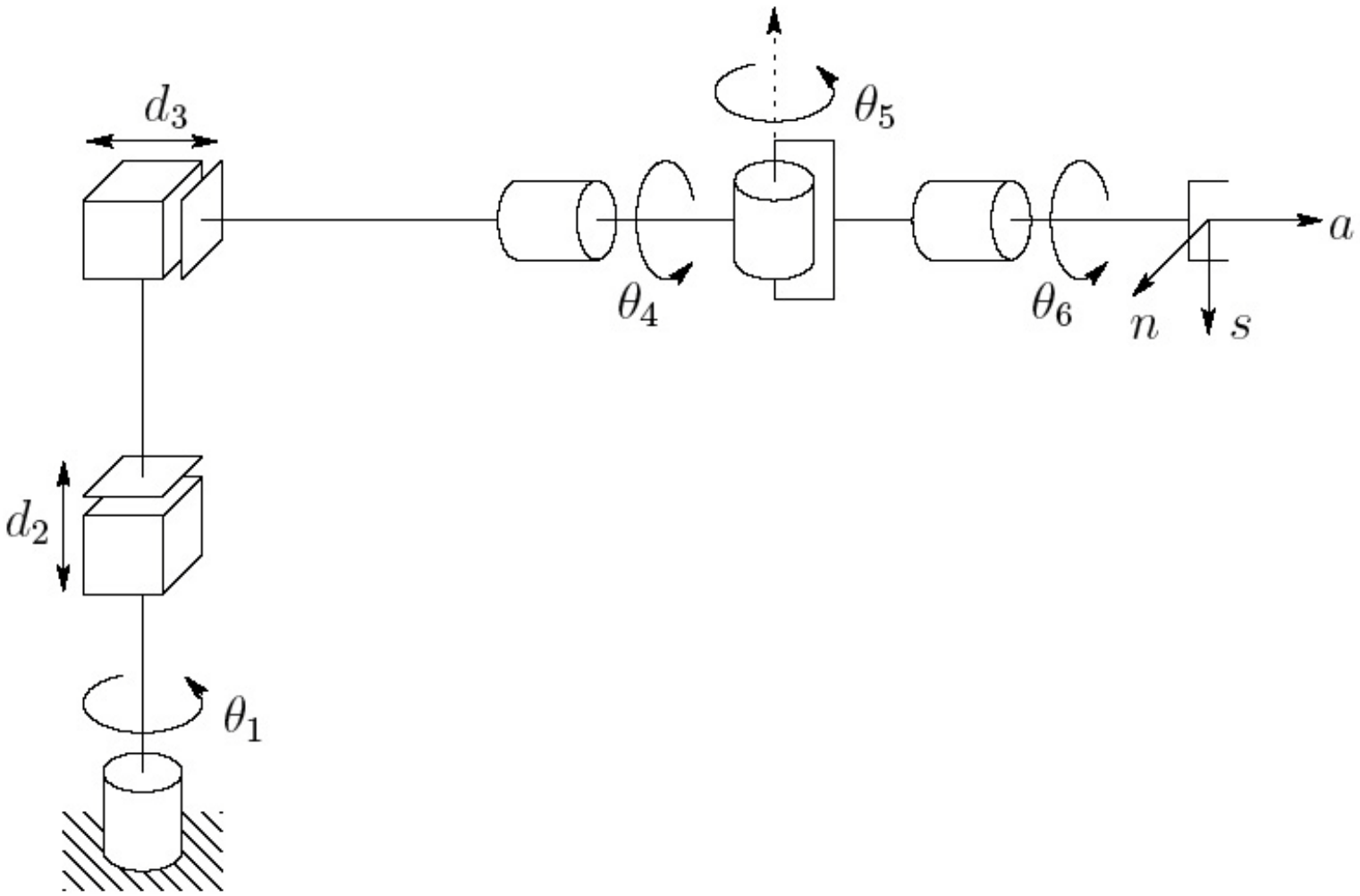


Figure 3.9: Cylindrical robot with spherical wrist.

RPP + Spherical Wrist

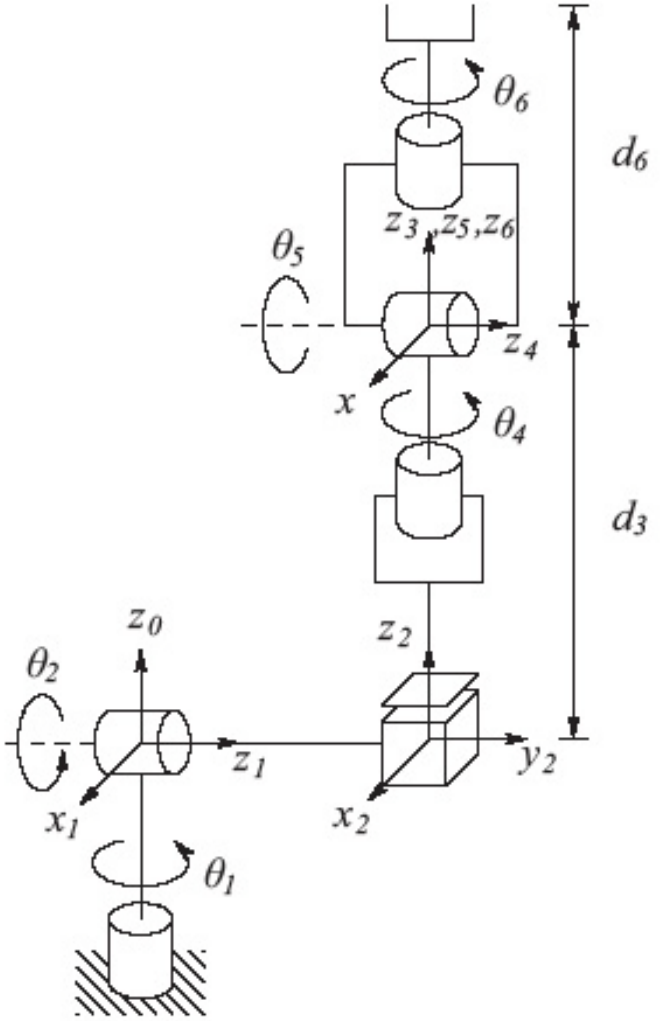
$$T_6^0 = T_3^0 T_6^3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11} = c_1 c_4 c_5 c_6 - c_1 s_4 s_6 + s_1 s_5 c_6$$

$$\vdots$$

$$d_z = -s_4 s_5 d_6 + d_1 + d_2$$

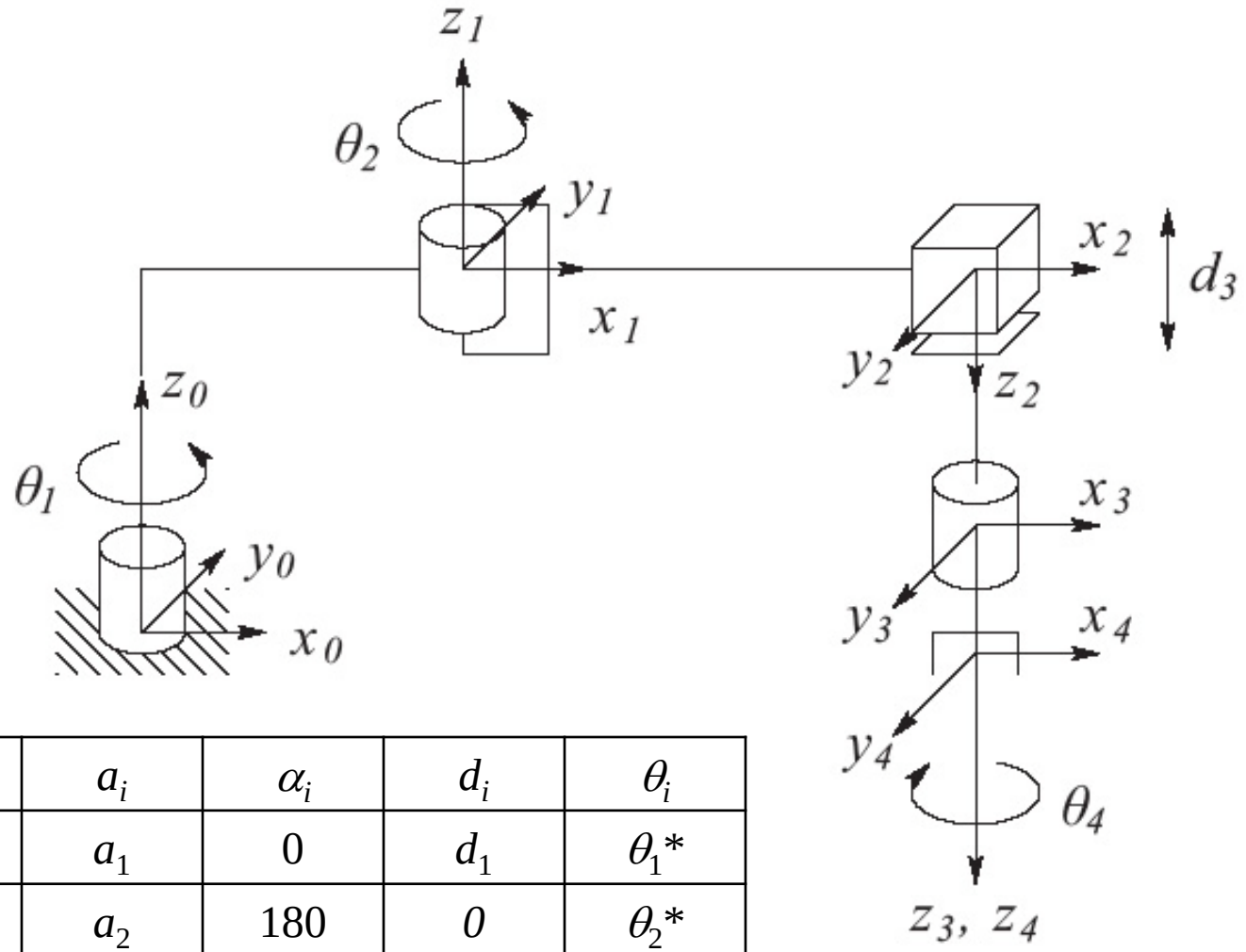
Stanford Manipulator + Spherical Wrist



Link	a_i	α_i	d_i	θ_i
1	0	-90	0	θ_1*
2	0	90	d_2	θ_2*
3	0	0	d_3*	0
4	0	-90	*0*	θ_4*
5	0	90	*0*	θ_5*
6	0	0	d_6	θ_6*

* joint variable

SCARA + 1DOF Wrist



Link	a_i	α_i	d_i	θ_i
1	a_1	0	d_1	θ_1*
2	a_2	180	*0*	θ_2*
3	0	0	d_3*	0
4	0	0	d_4	θ_4*

* joint variable